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Divisionalization in contests

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Abstract

To be represented by more than one contestant in a contest has advantages and disadvantages. This paper determines the conditions under which it is favorable to send several agents into the contest.

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1. Introduction

In many contest environments, the total number of players is limited, but players can choose between sending one or several agents to compete in a contest. For instance, in car races teams can send one or two cars into the race. In R&D contests, firms may have just one research laboratory or may divisionalize their research activities, installing several independent research units. More importantly, in markets in which price competition is effectively ruled out and all competition occurs via upfront sunk sales efforts, firms may decide to have only one sales department, or they may divisionalize and choose competing sales agencies. Two examples are the market for prescription drugs in countries in which price competition for drugs is effectively eliminated due to the indirect payment arrangements of the health care system, and the highly regulated pre-1994 insurance market in many European countries.[1] There seems to be little evidence for divisionalization in the market for prescription drugs, whereas, in the insurance market, some big firms owned several smaller insurance

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[1]For a description of the European insurance market see Rees and Kessner (1999).

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companies who contested for customers with each other and with the rest of the market and some companies even employed a number of sales agencies that cannibalized on each other.

In this paper we address the question whether divisionalization is profitable.[2] We disregard aspects such as intra-firm governance problems and competing divisions as an internal incentive system or economies of scale and other cost effects and concentrate on the strategic aspects that relate to the actual contest. There are three contest aspects. First, divisionalization increases the number of competitors. Typically this will reduce the rents earned by all contestants. Second, if a firm sends several competing agents into the contest, this firm's share in the aggregate rent is increased. Third, if a firm sends several agents into the contest and pays for their efforts, the divisions partially cannibalize on each other. The firm may take this into account and may reduce competition between the divisions.

In a related context, Baye et al. (1996) considered whether firms can gain from divisionalization in a Cournot oligopoly. The divisionalizing firm gains and overall profits in this industry are reduced. Divisionalization in contests has similar effects as regards total profits, but whether the divisionalizing firm gains depends on the number of competitors and on the characteristics of the contest.

2. Contests

Let $\mathcal{N}$ be the set of firms, with $\#\mathcal{N} = n$. Suppose that each of these firms has one contestant in a contest that makes an effort to win some prize of size B. Each contestant i chooses contest effort $x_i \in [0,\infty)$ which cannot be recovered, whether the contestant wins or not. Contest efforts determine contestants' probabilities q_i of winning the prize, according to a contest success function

$$q_i(x_1, \ldots, x_n) = \frac{(x_i)^a}{\sum_{j=1}^{n} (x_j)^a} \quad \text{for } a < n/(n-1). \tag{1}$$

This function has been suggested by Tullock (1980) and axiomatized by Skaperdas (1996). The coefficient a in (1) measures how much the contest outcome can be influenced by contest effort, and is called the *discriminatory power* of the contest success function. For instance, if $a \to 0$, each contestant has the same chance of winning, irrespective of contest efforts. If, instead, $a \to \infty$, (1) approaches a contest success function in which the contestant who makes the highest effort wins for sure. We limit the discriminatory power to $a \in [0, n/n-1)$ in order to have well-behaved optimization problems with equilibria in pure strategies and first-order conditions characterizing these equilibria.[3]

Firms are risk neutral. Suppose there is no incentive problem between firms and contestants. They maximize firms' payoffs

[2] Divisionalization can also be seen as a decision to delegate participation in a contest to multiple agents. This makes the analysis related to the literature on delegation in contests as in Baik and Kim (1997), Konrad et al. (1999) and Schoonbeek (1999) who all consider cases in which a principal delegates to one agent.

[3] For the equilibrium (in mixed strategies) for the case with $a > n/(n-1)$ see Baye et al. (1994).

$$\pi_i = q_i B - x_i. \tag{2}$$

Firm i wins the prize B with probability q_i and its contestant spends contest effort equal to x_i. The first-order condition for a payoff maximum and symmetry can be used to calculate the contest equilibrium efforts

$$x^*(n) = \frac{aB(n-1)}{n^2}, \tag{3}$$

with n being the number of contestants. The equilibrium probability of winning is $1/n$ for each contestant, yielding the equilibrium payoffs as

$$\pi^*(n) = \frac{B}{n} - \frac{aB(n-1)}{n^2}. \tag{4}$$

3. Fully non-cooperative divisions

Suppose firm n decides to divisionalize, i.e., to send two divisions to the contest and provides each division with incentives to maximize payoff as in (2). Divisionalization increases the set of contestants from $\{1, \ldots, n\} = \mathcal{N}$ to $\{1, \ldots, n_1, n_2\} = \mathcal{N}^d$. We denote $\{1, \ldots, (n-1)\} \equiv \mathcal{S}$ and $\{n_1, n_2\} \equiv \mathcal{D}$. The latter is the set of divisions of firm n. Divisionalization is profitable if the equilibrium payoff of the two divisions in the contest with $n+1$ participants exceeds the payoff of one participant in a contest with n participants. This is the case if $\pi^*(n) < 2\pi^*(n+1)$, or, using (4), if

$$\frac{(n+1)(n-1)n}{n^3 - n^2 + n + 1} > a. \tag{5}$$

Denote the left hand side of (5) by $a_0(n)$ and note that $a_0(n) \in (0, \frac{n}{n-1})$. Hence,

Proposition 1. *For a given total number n of firms there is a critical discriminatory power* $a_0(n) \in (0, n/n-1)$ *such that divisionalization by one firm is profitable for this firm if and only if* $a < a_0(n)$.

Intuitively, if the discriminatory power is very small, e.g., close to zero, then total effort becomes negligible in comparison to the prize. Accordingly, each firm earns almost B/n if it does not divisionalize, and almost $2B/(n+1)$ if it divisionalizes.

We also note that divisionalization reduces total industry profits, as $n\pi^*(n)$ in (4) is declining in n.

4. Cooperative divisions

If divisionalization occurs within a firm, the firm's divisions' efforts may be coordinated by a higher level decision making unit that takes into account that the contest effort in one division also reduces the other division's probability of winning.

In this case the total effect of divisionalization can be obtained analytically in two steps. First, suppose the new divisions behave non-cooperatively. This effect has been determined in Section 3. We call this the *team size effect* because it is based on the increased number of firm n's contestants. Second, divisions' choices of effort may be coordinated. We call this the *collusion effect*. To analyse the collusion effect we start with a situation in which all contestants in $\mathcal{N}^d$ behave non-cooperatively and ask if the two divisions n_1 and n_2 can increase their joint profits if they choose their efforts cooperatively.

If divisions maximize firm profit $(q_{n_1} + q_{n_2})B - x_{n_1} - x_{n_2}$, they take into account $(\partial q_i / \partial x_j) < 0$ for $i \neq j$, and therefore, coordination makes them reduce their effort compared to $x^*(n+1)$. This decrease is anticipated by contestants $k \in \mathcal{S}$. Let $\xi(\bar{x})$ be the symmetric non-cooperative equilibrium effort choices of divisions $n_i \in \mathcal{D}$, for given effort choices $x_j = \bar{x}$ for all $j \in \mathcal{S}$, and similarly, $\bar{\xi}(x)$ the symmetric non-cooperative equilibrium effort choices of all contestants $j \in \mathcal{S}$ for given effort choices $x_{n_i} = x$ for $n_i \in \mathcal{D}$. These functions could, in principle, be obtained analytically from solving the first-order conditions for fully non-cooperative firms,

$$\frac{a x_i^{a-1} \sum_{j \in \mathcal{N}^d \setminus \{i\}} x_j^a}{\left(\sum_{j \in \mathcal{N}^d} x_j^a\right)^2} B - 1 = 0, \tag{6}$$

for x as a function of $\bar{x}$, with x the non-cooperative effort level chosen by all contestants in $\mathcal{D}$ for given uniform effort levels $\bar{x}$ by all contestants $j \in \mathcal{S}$, and vice versa for $\bar{x}$ as a function of x. The intersection of these two curves characterizes the Nash equilibrium with $x_i = x_j = x^*(n+1) = aBn/(n+1)^2$ for all $i,j \in \mathcal{N}^d$ by (3). For $i \in S$, (6) can be rewritten as $a\bar{x}^{a-1}[(n-2)\bar{x}^a + 2x^a]B = [(n-1)\bar{x}^a + 2x^a]^2$. At the equilibrium value x^*, by differentiating this condition totally and making use of (3), the slope of $\bar{\xi}(x)$ can be obtained as

$$\bar{\xi}'(x) = \frac{2a(n-1)}{-n^2 - n - 2a + 2na}. \tag{7}$$

Lemma 1. $\bar{\xi}'(x) < 0$ *and* $\lim_{a \to 0} \bar{\xi}'(x) = 0$ *at* $x = \bar{x} = x^*(n+1)$.

For a proof of Lemma 1 observe that the numerator of (7) is positive and the denominator of (7) is negative for $a \leq (n/n-1)$ and converges toward $-\infty$ for $a \to 0$. The intuition for the limit property in Lemma 1 is as follows. By (6), for $a \to 0$ each contestant's marginal benefit from effort becomes infinitely small. Hence, a contestant would not like to spend much, even if other contestants spend huge amounts.

Using the envelope theorem and the fact that $(\partial \pi_i / \partial x_k) = -(1/n)$ for $i \neq k$ at the fully non-cooperative Nash equilibrium, the profit is higher for each division if they jointly reduce their contest effort x starting from $x = \bar{x} = x^*(n+1)$ if

$$-\frac{\mathrm{d}\pi_i}{\mathrm{d}x} = \frac{1}{n}(1 + (n-1)\bar{\xi}'(x)) > 0. \tag{8}$$

A joint decrease in their efforts increases their profits if the mutual direct effect of reduced effort among the divisions outweighs the equilibrium reactions by all other contestants. The inequality in (8) to be fulfilled is a necessary condition for collusion to be profitable. It resembles the condition that has been derived in Gaudet and Salant (1991) who consider merger in Cournot competition. The profitability effect of collusion is ambiguous in general. However, by Lemma 1, we have

Proposition 2. *A joint marginal anticipated reduction in contest effort among divisions is profitable if* (8) *holds*. (8) *holds if the discriminatory power of the contest is sufficiently low*.

The intuition for the result in Proposition 2 is as follows. Collusion on contest effort leads to a reduction in effort for the set of colluders. If this reduction in effort does not trigger an increase in other contestants' efforts, collusion is beneficial. As has been shown in Lemma 1, other contestants react by increases in their effort, but this reaction is very moderate if the discriminatory power of the contest is sufficiently low.

5. Conclusions

In this paper we consider the profitability of divisionalization in contests. We distinguish between a team size effect and a collusion effect and show that with high discriminatory power the team size effect of divisionalization reduces profits and the collusion effect is ambiguous, but both effects increase profits if the discriminatory power in the contest is low.

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